# Optimal Shape Design of Rotor Bar of Three-phase Squirrel Cage Induction Motor for NEMA Design D Torque-speed Characteristics

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Abstract — This paper presents a systematic optimal design algorithm to get a specific Class-D torque-speed characteristics for three-phase squirrel cage induction motor for the application to automatic valve actuator. In order to reduce computing time at the first stage of the optimization, the equivalent circuit method is combined with multi-objective particles swarm optimization (MOPSO) to reduce the design space. At the second stage of the optimization, time-stepping finite element method is combined with single-objective PSO to obtain an accurate optimal design.

### I. INTRODUCTION

Valve actuators that use an electric motor (usually three-phase induction motor) are called motor operated valve actuators (MOV). Recently, most of MOVs are used in heavy chemical industry, electrical power plant, gas, steel and ship building fields.

For general purpose, the induction motors are designed with a normal starting current, a normal torque-speed characteristics and low full-load slip. However, the threephase induction motors in MOVs require special load characteristics as shown in Fig. 1 which is different from general application. This kind of load requires maximum torque located at starting position. The motor with this torquespeed characteristic is called the National Electrical Manufacturer Association (NEMA) design D motor [1].

This kind of torque-speed characteristic can be obtained easily by using a wound-rotor induction motor with extra resistance inserted into the rotor. But wound-rotor induction motor is not a good choice for our application because of high price, complex structure and high failure rate. Therefore, it is necessary to propose a design method for three-phase squirrel cage induction motor to obtain design D torque-speed characteristic, even though the applications of this type of motor is very special.

In this paper, the systematic optimal design process is proposed to obtain NEMA design D torque-speed characteristic by regulating rotor bar geometry.

## II. OBJECTIVE FUNCTION FOR NEMA DESIGN D TORQUE-SPEED CHARACTERISTICS

In order to design an NEMA class D induction motor, it is necessary to define the objective function to estimate the design D torque-speed characteristics. Until now, there is seldom work focused on this aspect [2]. This paper represents a method to estimate a design D torque-speed characteristic based on equivalent circuit theory.

Fig. 2 shows the phase equivalent circuit of three phase induction motor. Here,  $V_1$  is phase voltage,  $R_1$ ,  $X_1$ ,  $X_m$ ,  $R_2$ 

and  $X_2$  are stator resistance, reactance, magnetizing reactance, rotor resistance and reactance respectively.

From the typical equivalent circuit theory the expression of mechanical torque can be obtained as follows:

$$T_{mech} = \frac{1}{\omega_s} \left\{ \frac{3V_{1,eq}^2 (\dot{R}_2 / s)}{(R_{1,eq} + (\dot{R}_2 / s))^2 + (X_{1,eq} + \dot{X}_2)^2} \right\}$$
(1)

where  $\omega_s$  is the synchronous speed.  $V_{l,eq}$  and  $Z_{l,eq}$  can be calculated as follows:

$$V_{1,eq} = \frac{jX_m V_1}{R_1 + j(X_1 + X_m)}$$
(2)

$$Z_{1,eq} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)}.$$
(3)

Substituting (2) and (3) to (1), the mechanical torque can be expressed as [3]:

$$T_{mech} = 1/(As + Bs^{-1} + C)$$
(4)

where the coefficients A, B and C are expressed as follows:

$$4 = \frac{((X_1 + X_2^{'})^2 + R_1^2)}{3R_2^{'}V_1^2} + \frac{2X_2^{'}(X_1^2 + R_1^2 + X_1X_2^{'})}{3R_2^{'}V_1^2X_m} + \frac{X_2^{'2}(X_1^2 + R_1^2)}{3R_2^{'}V_1^2X_m^2}$$
(5)

$$B = \frac{R_2}{3V_1^2} + \frac{2R_1X_1}{3V_1^2X_m} + \frac{R_2(X_1^2 + R_1^2)}{3V_1^2X_m^2}$$
(6)

$$C = \frac{2R_1}{3V_1^2}$$
(7)

The maximum torque slip can be obtained, from (4), as follows:

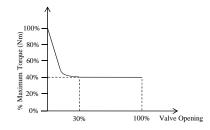


Fig. 1. Required valve load characteristics curve

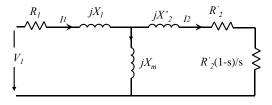


Fig. 2. Equivalent circuit of induction motor

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$$s_{\max} = \sqrt{B/A} \tag{8}$$

From (8), the maximum torque slip can be calculated by using equivalent circuit parameters. The parameters have direct relation with motor geometry.

When the stator of the motor is fixed, the equivalent circuit parameters are related only with the geometry of the rotor bar. So it is possible to regulate maximum torque slip to close to one by varying the rotor bar geometry. The maximum starting torque, furthermore, also has relation with the rotor bar geometry.

The NEMA design D torque-speed characteristics can be achieved by limiting the maximum torque slip close to one, and maximizing the starting torque simultaneously.

## III. OPTIMAL SHAPE DESIGN OF ROTOR BAR

The optimization procedure is summarized as follows:

Stage 1: Rough design

In this stage the design target is reducing the design space. The value of objective function can be calculated by using equivalent circuit of induction motor [4], [5]. The main degrees of freedom characterizing the shape of the rotor bar are  $w_1$ ,  $w_2$ ,  $w_3$ ,  $h_1$ ,  $h_2$  and  $h_3$  as shown in Fig. 3. And the ranges of design variables are listed in Table I. Thus the design problem for stage 1 can be treated as a vector, 6-parameter optimization problem shown as (9).

Maximize 
$$f_1(\mathbf{x}) = T_s$$
  
Minimize  $f_2(\mathbf{x}) = \left| 1 - \sqrt{B/A} \right|$  (9)  
subject to  $\mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U$ ,  $\mathbf{x} = [x_1 x_2 ... x_6]$ 

#### Stage 2: Fine design

In stage 2 the optimal target is obtaining accurate solution by using FEM. The design parameters are same as previous stage. And the reduced design space can be adopted to save computing time. Because the design D torque speed characteristic has already achieved in stage 1 so, in stage 2 the problem can be reduced to a single objec-

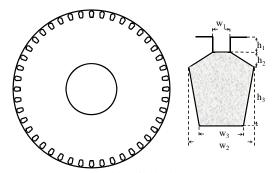


Fig. 3. Rotor and design variables of induction motor

TABLE I RANGE OF DESIGN VARIABLES

Variable	$w_I$	$W_2$	$W_3$	$h_{I}$	$h_2$	$h_3$
Unit	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]
Min	0.5	1.0	1.0	0.5	0.5	1.0
Max	3.0	5.2	4.0	1.5	1.5	12.0

tive optimization problem.

The chosen optimizers for this problem are multiobjective particle swarm optimization (MOPSO) in stage 1 and single-objective PSO in stage 2. The design process is summarized in Fig. 4.

After stage 1 the Pareto-optimal solutions are as shown in Fig. 5. For example, in the second stage optimization, the design space can be reduced based on the solution A, B or C.

In the version of full paper, the optimization procedure will be explained in more detail, and the torque-speed characteristics of the optimized design will be compared with experimentally measured ones.

#### IV. REFERENCES

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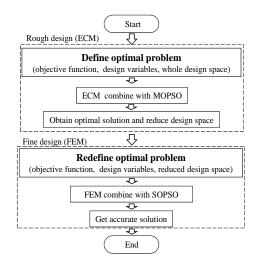


Fig. 4. Design process of design D induction motor

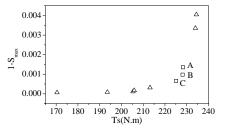


Fig. 5. Pareto-optimal solutions obtained by MOPSO